The intrinsic impedances for various materials have been examined earlier. They are repeated here for reference.

partially conducting medium: $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}}$

conducting medium: $\eta = \sqrt{\dfrac{\omega\mu}{\sigma}}\,\underline{/45^\circ}$

perfect dielectric: $\eta = \sqrt{\dfrac{\mu}{\epsilon}}$

free space: $\eta_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} \approx 120\pi\Omega$

EXAMPLE 3. Traveling **E** and **H** waves in free space (region *1*) are normally incident on the interface with a perfect dielectric (region *2*) for which $\epsilon_r = 3.0$. Compare the magnitudes of the incident, reflected, and transmitted **E** and **H** waves at the interface.

$$\eta_1 = \eta_0 = 120\pi\ \Omega \qquad \eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = 217.7\ \Omega$$

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = -0.268 \qquad \frac{H_0^r}{H_0^i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = 0.268$$

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_1 + \eta_2} = 0.732 \qquad \frac{H_0^t}{H_0^i} = \frac{2\eta_1}{\eta_1 + \eta_2} = 1.268$$

14.8 OBLIQUE INCIDENCE AND SNELL'S LAWS

An incident wave that approaches a plane interface between two different media generally will result in a transmitted wave in the second medium and a reflected wave in the first. The *plane of incidence* is the plane containing the incident wave normal and the local normal to the interface; in Fig. 14-5 this is the xz plane. The normals to the reflected and transmitted waves also lie in the plane of incidence. The *angle of incidence* θ_i, the *angle of reflection* θ_r, and the *angle of transmission* θ_t—all defined as in Fig. 14-5—obey *Snell's law of reflection,*

$$\theta_i = \theta_r$$

and *Snell's law of refraction,*

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}$$

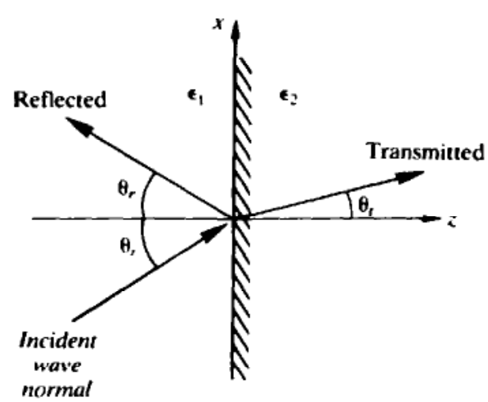

Fig. 14-5

EXAMPLE 4. A wave is incident at an angle of 30° from air to teflon, $\epsilon_r = 2.1$. Calculate the angle of transmission, and repeat with an interchange of the regions.

Since $\mu_1 = \mu_2$,

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sin 30^\circ}{\sin\theta_t} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = \sqrt{2.1} \qquad \text{or} \qquad \theta_t = 20.18^\circ$$

From teflon to air,

$$\frac{\sin 30^\circ}{\sin\theta_t} = \frac{1}{\sqrt{2.1}} \qquad \text{or} \qquad \theta_t = 46.43^\circ$$

Supposing both media of the same permeability, propagation from the optically denser medium ($\epsilon_1 > \epsilon_2$) results in $\theta_t > \theta_i$. As θ_i increases, an angle of incidence will be reached that results in $\theta_t = 90^\circ$. At this *critical angle* of incidence, instead of a wave being transmitted into the second medium there will be a wave that propagates along the surface. The critical angle is given by

$$\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

EXAMPLE 5. The critical angle for a wave propagating from teflon into free space is

$$\theta_c = \sin^{-1}\frac{1}{\sqrt{2.1}} = 43.64^\circ$$

14.9 PERPENDICULAR POLARIZATION

The orientation of the electric field **E** with respect to the plane of incidence determines the *polarization* of a wave at the interface between two different regions. In *perpendicular* polarization **E** is perpendicular to the plane of incidence (the xz plane in Fig. 14-6) and is thus parallel to the (planar) interface. At the interface,

$$\frac{E_0^r}{E_0^i} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

and

$$\frac{E_0^t}{E_0^i} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

Note that for normal incidence $\theta_i = \theta_t = 0^\circ$ and the expressions reduce to those found in Section 14.8.

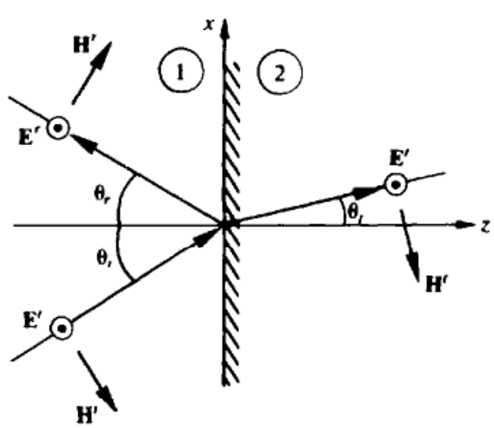

Fig. 14-6